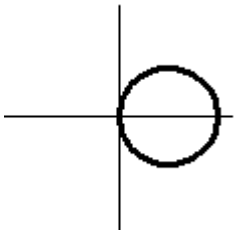
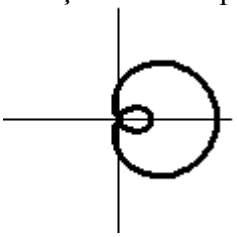
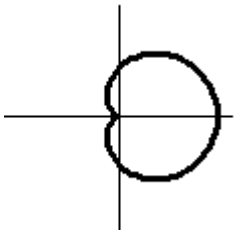
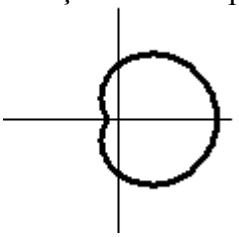
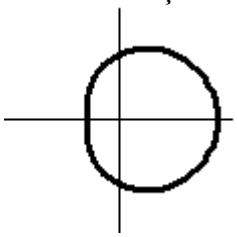
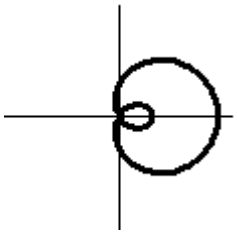
HOW TO GRAPH POLAR FUNCTIONS OF THE FORM $r = a \pm b\cos\theta$ OR $r = a \pm b\sin\theta$

1. Determine the shape of the graph by finding $\left|\frac{a}{b}\right|$.

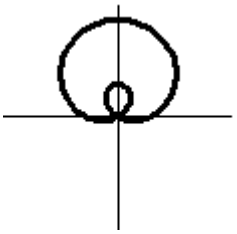
$\left\|\frac{a}{b}\right\| = 0$	$\left\|\frac{a}{b}\right\| < 1$	$\left\|\frac{a}{b}\right\| = 1$	$1 < \left\|\frac{a}{b}\right\| < 2$	$\left\|\frac{a}{b}\right\| \geq 2$
circle	limaçon with loop	cardioid	limaçon with dimple	convex limaçon
				

2. Determine the axis of symmetry by the trigonometric function used.

$r = a \pm b\cos\theta$
symmetric over polar axis ($x-$axis)

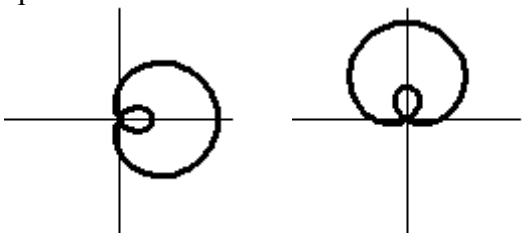


$r = a \pm b\sin\theta$
symmetric over $\theta = \frac{\pi}{2}$ ($y-$axis)

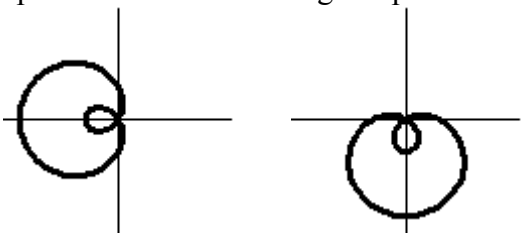


3. Determine the "direction" of the graph by the sign of b.

$b > 0$
"bigger" end on the right/top
"puckered" end on the left/bottom



$b < 0$
"bigger" end on the left/bottom
"puckered" end on the right/top



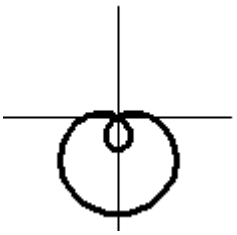
4. Determine the $x-$ and $y-$intercepts by finding the points corresponding to $\theta = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$

$\theta = 0, \pi$ correspond to positive and negative $x-$intercepts (assuming $r > 0$)
$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$ correspond to positive and negative $y-$intercepts (assuming $r > 0$)

If $r < 0$, the intercept is on the "other side" (negative vs positive, and vice versa) of the corresponding axis

Example: Graph $r = 2 - 3\sin\theta$

1. $\left|\frac{2}{-3}\right| < 1$ limaçon with loop



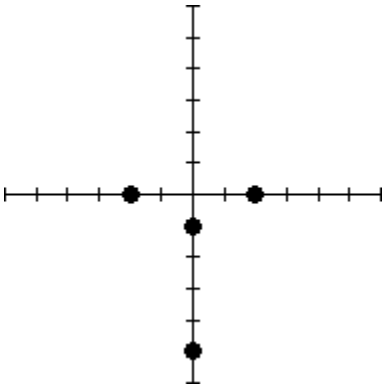
2. equation uses $\sin\theta$ symmetric over $\theta = \frac{\pi}{2}$ ($y-$axis)

3. $-3 < 0$ "bigger" end on the bottom, "puckered" end on the top

4.

θ	$r = 2 - 3\sin\theta$
0	2 (positive $x-$intercept)
$\frac{\pi}{2}$	–1 (negative $y-$intercept)
π	2 (negative $x-$intercept)
$\frac{3\pi}{2}$	5 (negative $y-$intercept)

Intercepts only



Complete graph